Quasi-Linearization Technique for Estimating Aircraft States from Flight Data

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This paper briefly describes a practical technique for obtaining weighted least-squares estimates of aircraft attitude angles, positions, and velocities from measurements recorded during routine flight tests. The measured accelerations and attitude rates are used as inputs to six-degree-of-freedom kinematic equations. Unknown parameters such as measurement biases and initial conditions are determined using a quasi-linearization identification method. This technique has been successfully used in a variety of flight test situations and with several types of aircraft. Particular results include trajectory smoothing during the transition and hover of V/STOL aircraft and the measurement of turbulence during the flight of a probe aircraft through the vortex wake of a large jet transport. Other types of results, briefly noted, include the estimation of instrument calibration terms, the determination of winds, the estimation of states not originally measured, and the determination of aerodynamic derivatives.

Introduction

THIS paper considers the problem of estimating aircraft states (positions, velocities, and attitude angles) from measurements recorded during flight. In many flight test situations, the measured quantities may contain significant inaccuracies (noise and bias) and it may be difficult to make a direct measurement of one or more of the aircraft states. This paper presents a solution to this problem by application of a quasi-linearization identification technique. Measurement bias terms and state initial conditions are treated as unknown parameters that are determined during the postflight processing on a general purpose digital computer. The computing technique provides a weighted least-squares estimate for the time history of the vehicle states.

Quasi-linearization identification methods have been used in several recent investigations[1-4] for estimating aircraft stability and control derivatives. These previous studies incorporated a mathematical model of the aircraft representing the inertial forces and moments acting on the vehicle. The technique in this paper uses no mathematical model of the aircraft, but rather uses the inertial accelerations and attitude rates directly measured by the airborne instrumentation. Some studies[5,6] have used inertial measurements (with Kálmán filtering methods) to smooth aircraft three-degree-of-freedom position data. Other studies[7,8] have used inertial measurements (with simple regression methods) to smooth three-degree-of-freedom longitudinal angle and position data. The technique in this paper, however, is formulated for the complete coupled six-degree-of-freedom motions and, further, is applied to a much broader variety of aircraft flight test problems.

This paper first outlines the computing technique and then illustrates the use of the technique in two example flight test situations. Other applications and further considerations are then noted.

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Description of Computing Technique

Figure 1 is a flow diagram of the computational procedure. The standard six-degree-of-freedom aircraft kinematic equations are programmed on the digital computer. Inputs to the kinematic equations are measured time histories of the linear accelerations (a_x', a_y', a_z') and the rotational rates (p', q', r'). The estimated time histories of output states from the kinematic equations include the three linear velocities $(\hat{u}, \hat{v}, \hat{w})$, the three Euler angles $(\hat{\theta}, \hat{\phi}, \hat{\psi})$, and the three linear positions $(\hat{x}, \hat{y}, \hat{h})$. Unknown parameters such as the initial conditions and bias terms are determined by an iterative parameter identification method (quasi-linearization) that seeks to minimize the weighted least-squares difference between the estimated set and the measured set (or partial set) of output states. The weighting functions on each of the output states are taken as $W_i = 1/\alpha_i^2$, where α_i represents the expected accuracy in the measurement of the ith state. If one or more of the output states are not measured, then the corresponding weight, W_i, is taken as zero.

Kinematic Equations

The aircraft kinematic equations programmed on the digital computer were

$$\dot{u} = a_x - qw + rv - g \sin\theta \quad (1)$$

$$\dot{v} = a_y - ru + pw + g \cos\theta \sin\phi \quad (2)$$

$$\dot{w} = a_z + qu - pv + g \cos\theta \cos\phi \quad (3)$$

$$\dot{\theta} = q \cos\phi - r \sin\phi \quad (4)$$

$$\dot{\phi} = q \tan\theta \sin\phi + r \tan\theta \cos\phi + p \quad (5)$$

$$\dot{\psi} = r \cos\phi/\cos\theta + q \sin\phi/\cos\theta \quad (6)$$

$$\dot{h} = u \sin\theta - v \cos\theta \sin\phi - w \cos\theta \cos\phi \quad (7)$$

$$\dot{x} = u \cos\theta \cos\psi + v(\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi) + w(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \quad (8)$$

$$\dot{y} = u \cos\theta \sin\psi + v(\sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi) + w(\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi) \quad (9)$$

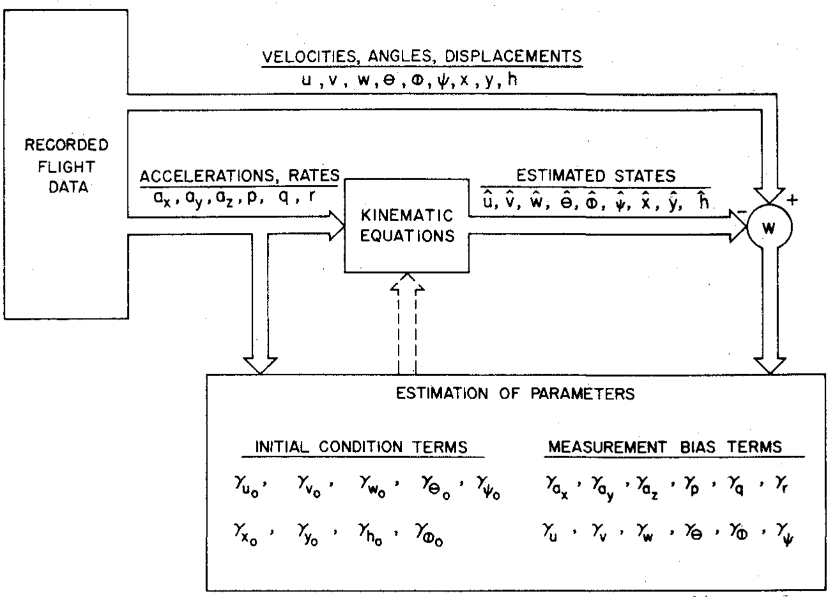

Fig. 1 Computing technique for estimating aircraft states.

Computer Solution

Define the problem as

$$\dot{s} = f(s,m), \quad s(0) = s_0 \tag{10}$$

where

$$s^T = (u,v,w,\theta,\phi,\psi,h,x,y) \tag{11}$$

$$m^T = (a_x,a_y,a_z,p,q,r) \tag{12}$$

and m includes constant bias terms in addition to the input time series data

$$m = m_b + m' \tag{13}$$

The initial condition terms s_0 along with measurement bias terms m_b and s_b are treated as a vector of unknown constant parameters

$$\gamma^T = (s_0{}^T, m_b{}^T, s_b{}^T)$$

or

$$\gamma^T = [\gamma(u)_0,\gamma(v)_0,\gamma(w)_0,\gamma(\theta)_0,\gamma(\phi)_0,\gamma(\psi)_0,\gamma(h)_0,\gamma(x)_0, \gamma(y)_0,\gamma(a)_x,\gamma(a)_y,\gamma(a)_z,\gamma_p,\gamma_q,\gamma_r,\gamma_u,\gamma_v,\gamma_w,\gamma_\theta,\gamma_\phi,\gamma_\psi] \tag{14}$$

which are to be determined such as to minimize the cost function

$$J = \frac{1}{2}\int_0^{t_f} \epsilon^T W \epsilon dt \tag{15}$$

where ϵ is a time-varying vector of output state residuals

$$\epsilon = s' - s - s_b \tag{16}$$

and W is a diagonal matrix (either constant or time varying). The minimization of J with respect to γ provides an optimum "smoothing" of the measured states and an optimum estimate of any other states not measured.

The problem of parameter optimization has been studied by many authors[9,10] and several methods could probably be used to search for the optimum parameter values. For this paper the method termed quasi-linearization or modified Newton-Raphson[11] was used. With this method, initial estimates are made for the set of parameter values and then these estimates are successively improved in the following iterative manner

$$\hat{\gamma}_{new} = \hat{\gamma}_{old} + [\int_0^{t_f} A^T W A dt]^{-1}\int_0^{t_f} A^T W \epsilon dt \tag{17}$$

where ϵ is the time-varying vector of the output state residuals and A is a matrix of time-varying influence functions representing the gradient of small variations in the output states with respect to the parameters γ. These influence functions are computed using an auxiliary set of equations linearized about the estimated states (old) in each iteration.

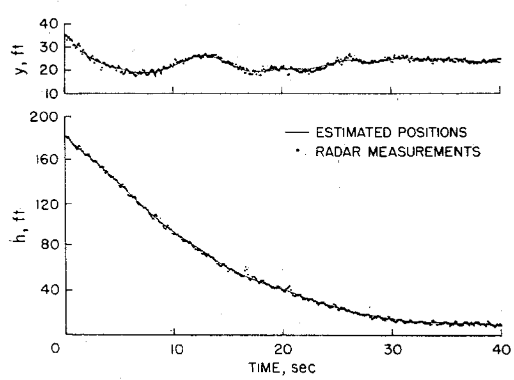

Fig. 2 Estimated positions compared with radar measurements for V/STOL landing approach.

Application Examples

This technique has been successfully applied in a variety of flight test programs and with several types of aircraft. Some of these examples are presented in this paper. These applications differ in the type of aircraft and maneuver, the number and quality of the measured output states, the form of weighting functions used with these states, and the number and type of parameters to be determined. The applications demonstrate several types of results: smoothing of the measured output states, estimation of states not originally measured, estimation of instrumentation bias terms, and the determination of winds and turbulence.

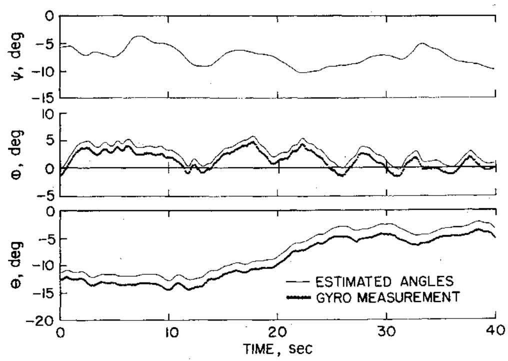

Fig. 3 Estimated angles compared with gyro measurements for V/STOL landing approach.

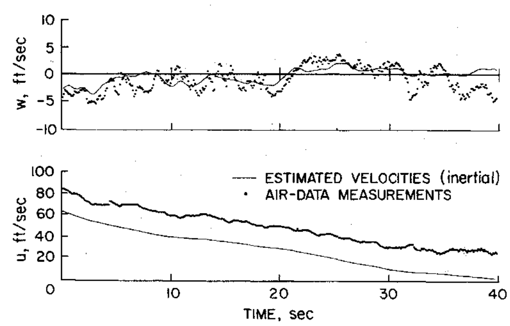

Fig. 4 Estimated velocities compared with air-data measurements for V/STOL landing approach.

Application to V/STOL Landing Approach

The flight test data used for this example were recorded during a routine transition to hover for the XV-5 experimental V/STOL aircraft.[12] The output states used for smoothing were the three positions (x,y,h) measured by ground-based radar and two attitude angles (θ,ϕ) measured by an onboard gyro (the angle ψ was not measured). The unknown parameters determined in the computer solution were the 9 initial condition terms, 3 accelerometer bias terms, 3 rate gyro bias terms, and 2 attitude gyro bias terms—a total of 17 parameters.

Figures 2-4 present some of the estimated states (solid lines) along with the measured data (small circles). These data represent a 40-sec portion of the transition maneuver from an initial altitude of 180-ft down to a hover condition about 10-ft above the runway. The results in Fig. 2 demonstrate excellent smoothing of the radar data. The estimated h and y positions are within the scatter of the radar measurements—about 2 ft rms in altitude and about 1 ft rms in crossrange. The results in Fig. 3 demonstrate smoothing of the measured attitude angles θ and ϕ. The constant difference between the measured and estimated attitude angles represents an estimate of the attitude gyro bias errors. This bias error of about 1° is within the expected offset in the vertical gyro. Although ψ was not measured, the computing technique does provide an estimate of this "missing state" (Fig. 3).

During this maneuver, the airflow velocities were measured by a pitot-static system and vanes. These measured air data include the components of wind as well as inertial velocities. For this particular example (Fig. 4), there was an approximate 20-fps headwind with gusty conditions. The difference between airflow velocities and the estimated inertial velocities provides a measure of the wind and turbulence acting on the airframe.

Application to Measurement of Wake Vortices

This next example illustrates a situation in which the measure of turbulence is a primary objective of the flight test. In this example, however, measurements of the aircraft position are not available and a somewhat different use of the weighting functions must be considered. This application uses data recorded during the flight of a probe aircraft (Lear-Jet) through the vortex wake of large jet transport.[13] The object of this flight test maneuver is to measure the magnitude of the vortex velocities within the wake as a function of the probe aircraft position. The output states used for smoothing were the three attitude angles (θ,ϕ,ψ) measured by gyros and the three velocities (u,v,w) measured by air data. The unknown parameters determined in the computer solution were 6 initial condition terms, 3 accelerometer bias terms, 3 rate gyro bias terms, and 3 attitude gyro bias terms—a total of 15 parameters.

The weighting functions on the velocity measurements were taken as time dependent. During that portion of the maneuver when the probe aircraft is within (or near) the vortex wake, these weighting functions were taken as zero. At other times when the probe aircraft was out of the vortex wake, these weights were taken as constant values. This choice of weightings, in effect, is an attempt to "match" those portions of measured velocity that are not disturbed by turbulence.

Estimated velocities during a representative maneuver are illustrated in Fig. 5. During the center portion of this maneuver, the probe aircraft is within the vortex wake. The difference between the measured air data and the estimated inertial velocity represents the vortex wake velocities. The magnitude of this vortex wake (in the y-h plane) is illustrated in the upper part of Fig. 6.

The aircraft positions during the maneuvers were not directly measured; however, the computing technique of this paper does allow an estimate of changes in the aircraft positions. The estimated position time histories (as shown in the bottom of Fig. 6) provide information on the relative position of the aircraft during the vortex measurements.

Further Applications and Considerations

Four additional examples will be discussed next to illustrate some of the other areas where this technique can be used in analysis of flight test data. The last two examples represent useful extensions to the basic equations outlined previously.

Measurement of Aerodynamic Derivatives

One of the primary uses for this computing technique has been to obtain good estimates of the vehicle velocities and attitude angles for use in the determination of the

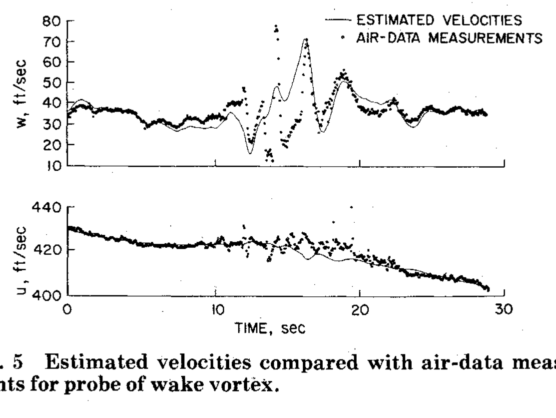

Fig. 5 Estimated velocities compared with air-data measurements for probe of wake vortex.

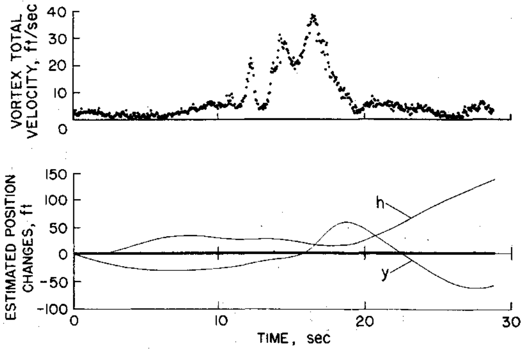

Fig. 6 Measured wake vortex velocity with esimated positions.

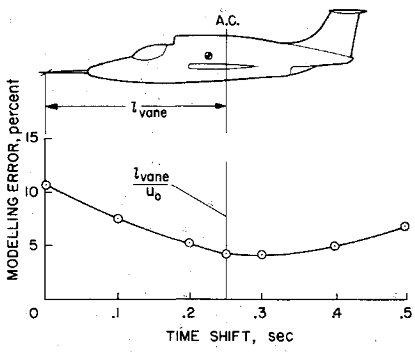

Fig. 7 Effect of shifting the time history of measured turbulence in aircraft parameter identification.

aircraft performance, stability, and control derivatives. Also, this technique provides a measurement of the winds and turbulence acting on the airframe, thus allowing a possible avenue of approach for the determination of aerodynamic derivatives during turbulent flight conditions (essentially all aircraft modeling to date has been done under strict, clear air conditions).

Some preliminary applications to the determination of aerodynamic derivatives have been made using data recorded during turbulent flight. An important consideration has been to determine the manner in which the measured turbulence interacts with the airframe. The turbulence, as measured by the vane forward of the nose, will interact in a time-varying manner along the length of the airframe. A cursory examination has shown (Fig. 7) that the aircraft modeling error can be minimized (i.e., a best match is obtained) if the measured turbulence is delayed by that amount of time required to travel from the vane to the effective aerodynamic center. Preliminary results are encouraging and indicate that aerodynamic derivatives may be obtained from routine turbulent flight conditions.

Use of Inertial Platform Measurements

In the applications discussed so far, the body-fixed airframe rate gyros and accelerometers have been used in the manner of a strapped-down inertial measuring unit. The estimation technique can be used as well with measurements from an inertial platform such as in a local vertical orientation.

One such application is to simply redefine the states in Eqs. (1-6) to represent motions of the platform, rather than the motions of the aircraft. In a typical application, the measured inputs are the three accelerations a_x, a_y, a_z, with respect to the platform axes (the platform attitude rates, p, q, r, are generally quite small and are not directly measured). The interpretation of the results from this application is: the parameters γ_p, γ_q, γ_r represent estimates of the platform drift rates; the states θ, ϕ, ψ represent estimates of platform angles; and the states u, v, w represent estimates of the velocities with respect to the platform. The estimated positions h, x, y in this application have the same interpretation as before.

Use of Additional Measurement Sources

Although the basic formulation, Eqs. (1-16), can be used in a large variety of flight test situations, there are certain instances when it is quite useful to expand the problem statement. One extension, for example, is to include position measurements from several sources. In many flight test situations the position measurements may include data from sources such as multiple radar sites, optical tracking, radio and barometric altimeters, etc.

The additional measurements h_1', x_1', y_1', ... can be incorporated into the computing algorithm by expanding the cost functional [Eqs. (15) and (16)] with additional terms of the form

$$\epsilon^T = (\ldots, h_1' - \hat{h} - \gamma_{h_1}, x_1' - \hat{x} - \gamma_{x_1}, y_1' - \hat{y} - \gamma_{y_1}, \ldots)$$

The weighting functions associated with each of the measuring sources may be time varying to account for changing accuracy of the measurements along the trajectory. A further extension, requiring additional state equations, however, is to formulate the new states in the coordinate system of the basic position measurements, such as radar range, tracking angles, etc. The states defined in this manner provide a more straightforward use of the weighting functions (e.g., W may be constant) and allow the determination of bias terms in some of the position measuring systems.

By increasing the problem dimensions to account for multiple measurement sources, the estimated states will incorporate all information, using weighted least squares for a given flight test situation.

Addition of Higher-Order Instrument Calibration Terms

The only types of instrument errors considered in the basic Eqs. (1-16) were bias terms. Additional types of error sources such as accelerometer and rate-gyro scale factors have been implemented and found quite useful, particularly when there are some indications that these instrumentation calibrations are not well known (or vary in-flight). One is tempted to increase the number of calibration terms, that is, include higher-order scale factors, sensor cross-coupling errors, etc. However, there are practical limitations on the number of unknown parameters that can be identified in each flight test situation. The relative variance in the estimates of the parameters[14] can be obtained from the covariance matrix

$$[\int_0^{t_f} A^T W A dt]^{-1}$$

[see Eq. (17)]. The addition of parameters that are weakly identifiable or linearly dependent with other parameters will increase expected variance on the parameter estimates.[15] (The addition of parameters that are not identifiable will cause the matrix

$$[\int_0^{t_f} A^T W A dt]$$

to be singular and no computer solution is possible.)

One solution to the problem of estimating parameters that are weakly identifiable is to incorporate prior information in the cost function[11]

$$J = \frac{1}{2}(\hat{\gamma} - \gamma_0)^T Q(\hat{\gamma} - \gamma_0) + \frac{1}{2}\int_0^{t_f} \epsilon^T W \epsilon dt$$

where γ_0 is the vector of a priori estimates for the parameter values and Q is a matrix of weighting functions which expresses the relative confidence in these prior estimates. The iterative estimation procedure, from Eq. (17), now becomes

$$\hat{\gamma}_{\text{new}} = \hat{\gamma}_{\text{old}} + [Q + \int_0^{t_f} A^T W A dt]^{-1}[Q(\gamma_0 - \hat{\gamma}_{\text{old}}) + \int_0^{t_f} A^T W \epsilon dt]$$

This extension to the computing technique has heen found to be useful both for flight test situations where there are a large number of parameters to be determined and for situations where the dynamic motions are mild; that is, where all the dynamic states are not strongly excited and there is a tendency to have large variances in some of the parameter estimates.

Concluding Remarks

This paper has presented some results based on a technique for obtaining weighted least-squares estimates of the aircraft states from recorded flight test data. This technique, in effect, uses the airframe rate gyros and accelerometers in the manner of a strapped-down inertial platform. The measurement bias terms and state initial conditions are treated as unknown parameters that are determined during the postflight processing on a general purpose digital computer.

Applications presented in this paper illustrate the use of this technique for different types of aircraft and maneuvers. The primary results include 1) smoothed time histories of measured positions, velocities, and attitude angles, 2) estimated time histories of the other positions, velocities, and attitude angles not originally measured, 3) estimates of instrumentation bias terms, and 4) measured time histories of winds and turbulence. Because these types of data are required in a variety of flight test programs, the technique outlined in this paper should find wide application.

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